

Flavour-symmetry correction to the “naïve” Zweig rule for the scalar-meson flavour-singlet

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Abstract

We present flavour-symmetric couplings for the OZI-allowed three-meson vertices of effective meson theories, which for the case of the two-meson channels to which the flavour-singlet scalar meson couples, are endowed with a correction factor with respect to the standard formula.

1 Introduction

Three-meson vertices give rise to the most important strong interactions which are considered by effective meson theories, as they reflect the simple fact that by quark pair creation mesons couple to pairs of mesons according to processes of the form

$$C \longleftrightarrow A + B \quad . \quad (1)$$

Within multiplets of SU_3 -flavour or U_3 -flavour symmetry, the relative magnitudes of the transition amplitudes for processes (1) are given by

$$\lambda \text{Tr} \left(\mathcal{M}_A \mathcal{M}_B \mathcal{M}_C^T \pm \mathcal{M}_B \mathcal{M}_A \mathcal{M}_C^T \right) \quad , \quad (2)$$

where \mathcal{M}_X is the 3×3 flavour matrix for meson X . It is understood in formula (2) that either the symmetric or the antisymmetric trace is to be taken, depending on the sign of the product of the three charge-conjugation quantum numbers. This way, charge conjugation and G -parity are automatically preserved.

Relation (2) is often referred to as the Zweig rule, since it fully suppresses exactly those two-meson decay modes which do not meet Zweig's criteria for meson-pair decay as given in Ref. [1] and moreover agrees with the quark line rules for Quantum Chromodynamics in the limit of large N_c as developed in Ref. [2].

The constant λ in front of expression (2) can be adjusted to experiment for each different set of three U_3 -flavour nonets (or octets and singlets in the case of SU_3 -flavour), such that the interaction Lagrangian for the theory may contain a long sum of all possible three-meson vertices, each with a different coupling constant. However, in the absence of a prescription for the relative intensities among the thus occurring terms in the interaction Lagrangian, one easily overlooks (see e.g. Ref. [3]) an inconsistency in the procedure for the scalar flavour-singlet two-meson transition modes if a unified coupling is introduced, as we will explain in the following. Note that, especially in the case of the scalar mesons, the employment of truly flavour-independent couplings may be crucial for the obtainment of reliable predictions in unitarised meson models, due to the large and highly nonlinear coupled-channel effects in these systems [4] (see also [5]).

2 All three-meson vertices and specific examples

In Ref. [6] we describe how the various values for the coupling constants λ can be unified. Our strategy becomes manageable, if we assume equal effective quark masses in the harmonic-oscillator expansion, resulting in a finite, albeit large, number of possible meson-pair channels

for each type of meson, which are all just given by the recoupling of the four involved valence (anti)quarks. As described in Ref. [6], the coupling constants boil down to the following expression

$$g \text{Tr} \left\{ \mathcal{M}_A \mathcal{M}_B \mathcal{M}_C^T \langle J, L, S, N; (j, \ell, s, n; A), (j, \ell, s, n; B) | \mathcal{P} | (J, \ell, s, n; C) \rangle + \right. \quad (3) \\ \left. + \mathcal{M}_B \mathcal{M}_A \mathcal{M}_C^T \langle J, L, S, N; (j, \ell, s, n; A), (j, \ell, s, n; B) | \bar{\mathcal{P}} | (J, \ell, s, n; C) \rangle \right\},$$

where g is universal, i.e., the same for all possible three-meson vertices. The quantum numbers J , L , S and N in formula (3) represent the total angular momentum, the relative orbital angular momentum, the total spin, and the relative radial excitation of the $A + B$ two-meson channel, respectively, whereas $(j, \ell, s, n; X)$ represent the corresponding quantum numbers for the $q\bar{q}$ -system that describes meson X . \mathcal{P} represents the exchange operator for quarks and $\bar{\mathcal{P}}$ for antiquarks. The matrix elements

$$\langle J, L, S, N; (j, \ell, s, n; A), (j, \ell, s, n; B) | \mathcal{P} | (J, \ell, s, n; C) \rangle \quad (\text{a})$$

$$\text{and} \quad \langle J, L, S, N; (j, \ell, s, n; B), (j, \ell, s, n; A) | \bar{\mathcal{P}} | (J, \ell, s, n; C) \rangle \quad . \quad (\text{b}) \quad (4)$$

determine the relative coupling constants for the various OZI-allowed three-meson vertices. They result from Fermi statistics applied to the valence (anti)quarks and Bose statistics to the meson pair in the four-particle harmonic oscillator expansion, which is not be confused with either quark dynamics or quark wave functions. Expressions (4a) and (4b) are equal, up to a factor ± 1 , depending on the sign of the product of the three charge-conjugation quantum numbers, which is equivalent to the choice of sign in formula (2). Total spin J , parity and charge conjugation are conserved and the OZI-rule ([1] and [7]) is respected by formula (3). The recoupling scheme is outlined in Ref. [6], whereas more details on the evaluation of the recoupling matrix elements (4) can be found in Ref. [8].

In order to make our point, instead of exhibiting all details of the calculation, we just give the results for three cases: the two-meson transitions of pseudoscalar, vector, and scalar mesons. Table 1 shows the nomenclature we used for the relevant mesons in this paper. The squares of the transition amplitudes to all channels which couple within our procedure are given in Table 2 for pseudoscalar mesons, in Table 3 for vector mesons, and in Table 4 for scalar mesons. In order to keep the tables as condensed as possible and since we assume that isospin is indeed a perfect symmetry, we may represent all members of an isomultiplet by the same symbol (t for isotriplet, d for isodoublet, 8 for the isosinglet flavour-octet member, and 1 for the flavour singlet)

Now, let us just analyse one horizontal line of one of the three tables, to make sure that the reader understands what the numbers stand for. Let us take the fourth line of Table 2. In the first column, under A , we find P for meson A , which hence characterises a meson out of the lowest-lying pseudoscalar nonet. In the second column, under B , we similarly find that meson B represents a meson out of the lowest vector nonet. In the third column, we find the quantum numbers for the relative motion of A and B , i.e., P -wave ($L = 1$) with total spin one ($S = 1$), in the lowest radial excitation ($N = 0$). Since the table refers to two-meson transitions of the lowest pseudoscalar meson nonet (P , indicated in the top of the table), the next four columns refer to its isotriplet member, which is the pion. We then find that the pion couples with a strength $\sqrt{1/6}$ to the tt (isotriplet-isotriplet) channel, which, following the above-discussed particle assignments for A and B , i.e., pseudoscalar and vector respectively, represents the $\pi\rho$ channel. Following a similar reasoning, we find that the pion couples with a strength $\sqrt{1/12}$ to KK^* . The total coupling of a pion to pseudoscalar-vector channels is given in the column under T by $\sqrt{1/4}$, which is the square root of the quadratic sum of the two previous couplings, i.e., $\sqrt{1/6 + 1/12}$.

The next set of coupling constants refer to the two-meson transitions of a kaon. We find $\sqrt{1/8}$ to td , which represents both of the possibilities pseudoscalar (isotriplet) + vector (isodoublet), i.e., πK^* , and pseudoscalar (isodoublet) + vector (isotriplet), i.e., $K\rho$, each with half the intensity given in the table. Next, we find in the table that the kaon couples with $\sqrt{1/8}$ to $d8$, which represents both of the possibilities pseudoscalar (isodoublet) + vector (SU_3 -octet isoscalar), i.e., $K + \text{octet-mixture of } \omega \text{ and } \phi$, and pseudoscalar (SU_3 -octet isoscalar) + vector (isodoublet), i.e., octet-mixture of η and $\eta' + K^*$, each with half the intensity given in the table. The kaon does not couple to the $d1$ channels in pseudoscalar + vector, which represent the channels with one isodoublet and one SU_3 singlet. The total coupling for the kaon to its pseudoscalar + vector channels sums up to $\sqrt{1/4}$, as one verifies in the column under T . The next two sets of coupling constants similarly refer to the two-meson transition modes of the isoscalar, either SU_3 -octet or SU_3 -singlet, partners of the pseudoscalar nonet.

A remark is here in place: When we identify the flavour isotriplet members of the lowest lying harmonic oscillator state with pions, the isodoublet states with kaons and so on, then we have actually in mind that the corresponding coupling constants of their three-meson vertices are to be folded in a coupled channel model where the real mesons come out as bound states and resonances (see e.g. [3], [9] and [10]). Hence, the above particle identification should not be taken too literally. At best, one may identify the harmonic oscillator states with objects which do not really exist in Nature, the so-called bare hadrons, *i.e.* valence $q\bar{q}$ -systems which are forbidden to couple to two-meson channels by means of valence quark pair creation. Nevertheless, the

unification of the coupling constants can only be achieved by taking into account the internal structure of the three mesons involved, for which here we have chosen harmonic oscillators.

3 A closer look at the results

From the three tables we may notice the following:

1. The intensities (couplings squared) for all strong two-meson transition modes of the pion (columns 4 to 7 in Table 2) add up to 1 (number at the very bottom of the eighth column); and the same result holds for the couplings to the two-meson channels of all other pseudoscalar nonet members, as well as for vector and scalar mesons (Tables 3 and 4). The reason for this property is the wave-function normalisation for the recoupling matrix elements of formula (3), which this way translates the flavour independence of strong interactions, very recently reconfirmed by experiment [11].

2. The subtotals (columns under T) for the strong two-meson transition intensities of the octet members are equal for each different mode (one horizontal line in each of the tables). This translates SU_3 -flavour independence of the strong interactions.

3. The subtotals of the flavour-singlet pseudoscalar and vector mesons are either twice as large as those of the flavour-octet members, or zero, in such a way that in both cases the total intensity adds up to 1. Unfortunately, the tables for axial vectors, tensors, etc. are too long to be shown here in a manageable form. Nevertheless, let us just mention that for all higher quantum numbers we find similar factors two and zero for the flavour-singlet couplings, with only one exception: the scalar mesons (Table 4).

4. If one uses formula (2), one similarly obtains these factors two and zero. However, in this case the same applies to scalar-meson transitions, *contrary* to our findings.

In our procedure (formula (3)), we thus find full flavour independence for all strong two-meson transitions of all mesons, whereas with formula (2) the flavour-singlet scalar couples twice as strong to its two-meson channels.

This feature of the three-meson couplings can only be fully appreciated once *all* two-meson channels (open and closed) are taken into account, which is much easier when their number is finite, and which takes a particularly manageable form in the harmonic-oscillator expansion for equal effective quark masses.

4 Conclusions

The flavour-singlet of scalar mesons has the quantum numbers of the vacuum ($|0\rangle$), which we also believe to be the quantum numbers of the valence $q\bar{q}$ -pair created in OZI-allowed strong two-meson transitions. Now, in general, the normalised sum of two orthonormal states $|\phi\rangle$ and $|0\rangle$ is given by $(|\phi\rangle + |0\rangle)/\sqrt{2}$. However, when $|\phi\rangle = |0\rangle$ (in which case they are not orthogonal), then the correctly normalised sum is given by $(|\phi\rangle + |0\rangle)/2$. This is exactly the reason for the extra factor $1/\sqrt{2}$ which we find with our procedure. Hence, we propose to modify formula (2) into

$$\lambda \frac{\text{Tr}(\mathcal{M}_A \mathcal{M}_B \mathcal{M}_C^T \pm \mathcal{M}_B \mathcal{M}_A \mathcal{M}_C^T)}{\sqrt{1 + \langle C | \text{flavour-singlet scalar meson} \rangle}} \quad , \quad (5)$$

in order to restore universal flavour independence for the three-meson vertices of effective theories for strong interactions.

5 Epilogue

Finally, we should mention that the Zweig rule for strong decays does not necessarily imply that singlets couple twice as strongly as octet members. On the contrary, the couplings for three-meson vertices for both subsets of the flavour nonet may be chosen independently in SU_3 -flavour-symmetric theories.

Moreover, we do neither assume here that the numbers of the three tables (2, 3 and 4) are the rigorously correct relative intensities for three-meson vertices, nor that the number of two-meson channels must be finite. For that, both the limit of equal effective valence quark masses and the harmonic oscillator expansion are probably too crude approximations. Those numbers are principally meant to pinpoint the scalar flavour-singlet problem. Nevertheless, in the light of the promising results of Ref. [10], in which works a unified coupling for pseudoscalars, vectors and scalars has been applied, the tables gain some credibility. Furthermore, the anti-De Sitter geometry [12], which has recently revived [13] as a possible candidate for quark confinement [14], is well approximated by a harmonic oscillator of universal frequency. This might provide an additional justification for the here employed harmonic-oscillator expansion, as not just a purely mathematical tool.

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symbol	$(n+1)^{2s+1}\ell_J$	J^{PC}
P	1^1S_0	0^{-+}
P'	2^1S_0	0^{-+}
V_0	1^3S_1	1^{--}
V_0'	2^3S_1	1^{--}
V_2	1^3D_1	1^{--}
S	1^3P_0	0^{++}
T	1^3P_1	1^{++}
U	1^1P_1	1^{+-}

Table 1: Nomenclature of mesonic $q\bar{q}$ systems relevant to this paper. The columns respectively contain our notation for the mesons, the $q\bar{q}$ quantum numbers (n is radial quantum number, s is total spin, ℓ is orbital and J is total angular momentum), and the more common quantum numbers (J , parity $P = (-1)^{\ell+1}$, and charge conjugation $C = (-1)^{\ell+s}$).

decay products A B rel. LSN			flavour channels and totals for P																			
			SU_3 -octet members															SU_3 singlets				
			isotriplets (t)					isodoublets (d)					isoscalars (8)					(1)				
tt	dd	$t8$	$t1$	T	td	$d8$	$d1$	T	tt	dd	88	11	T	tt	dd	88	11	T				
P	S	000	-	$\frac{1}{24}$	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{144}$	$\frac{1}{18}$	$\frac{1}{8}$	$\frac{1}{24}$	$\frac{1}{72}$	$\frac{1}{72}$	$\frac{1}{18}$	$\frac{1}{8}$	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{4}$	
V_0	U	000	-	$\frac{1}{24}$	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{144}$	$\frac{1}{18}$	$\frac{1}{8}$	$\frac{1}{24}$	$\frac{1}{72}$	$\frac{1}{72}$	$\frac{1}{18}$	$\frac{1}{8}$	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{4}$	
V_0	T	000	$\frac{1}{6}$	$\frac{1}{12}$	-	-	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$	-	$\frac{1}{4}$	-	$\frac{1}{4}$	-	-	$\frac{1}{4}$	-	-	-	-	-	
P	V_0	110	$\frac{1}{6}$	$\frac{1}{12}$	-	-	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$	-	$\frac{1}{4}$	-	$\frac{1}{4}$	-	-	$\frac{1}{4}$	-	-	-	-	-	
V_0	V_0	110	-	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{9}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{72}$	$\frac{1}{9}$	$\frac{1}{4}$	$\frac{1}{12}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{9}$	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{2}{9}$	$\frac{1}{18}$	$\frac{1}{18}$	$\frac{1}{2}$	
							1					1					1					1

Table 2: Transition intensities for the coupling of pseudoscalar mesons to meson pairs. The interpretation of the content of the table is explained in the text.

decay products A B rel. LSN			flavour channels and totals for V_0																				
			SU_3 -octet members															SU_3 singlets					
			isotriplets					isodoublets					isoscalars					(1)					
			(t)					(d)					(8)					(1)					
			tt	dd	$t8$	$t1$	T	td	$d8$	$d1$	T	tt	dd	88	11	T	tt	dd	88	11	T		
P	U	010	-	$\frac{1}{72}$	$\frac{1}{108}$	$\frac{1}{54}$	$\frac{1}{24}$	$\frac{1}{48}$	$\frac{1}{432}$	$\frac{1}{54}$	$\frac{1}{24}$	$\frac{1}{72}$	$\frac{1}{216}$	$\frac{1}{216}$	$\frac{1}{54}$	$\frac{1}{24}$	$\frac{1}{36}$	$\frac{1}{27}$	$\frac{1}{108}$	$\frac{1}{108}$	$\frac{1}{12}$		
P	T	010	$\frac{1}{18}$	$\frac{1}{36}$	-	-	$\frac{1}{12}$	$\frac{1}{24}$	$\frac{1}{24}$	-	$\frac{1}{12}$	-	$\frac{1}{12}$	-	-	$\frac{1}{12}$	-	-	-	-	-		
V_0	U	010	$\frac{1}{18}$	$\frac{1}{36}$	-	-	$\frac{1}{12}$	$\frac{1}{24}$	$\frac{1}{24}$	-	$\frac{1}{12}$	-	$\frac{1}{12}$	-	-	$\frac{1}{12}$	-	-	-	-	-		
V_0	T	010	-	$\frac{1}{18}$	$\frac{1}{27}$	$\frac{2}{27}$	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{1}{108}$	$\frac{2}{27}$	$\frac{1}{6}$	$\frac{1}{18}$	$\frac{1}{54}$	$\frac{1}{54}$	$\frac{2}{27}$	$\frac{1}{6}$	$\frac{1}{9}$	$\frac{4}{27}$	$\frac{1}{27}$	$\frac{1}{27}$	$\frac{1}{3}$		
S	V_0	010	-	$\frac{1}{24}$	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{144}$	$\frac{1}{18}$	$\frac{1}{8}$	$\frac{1}{24}$	$\frac{1}{72}$	$\frac{1}{72}$	$\frac{1}{18}$	$\frac{1}{8}$	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{4}$		
P	P	100	$\frac{1}{36}$	$\frac{1}{72}$	-	-	$\frac{1}{24}$	$\frac{1}{48}$	$\frac{1}{48}$	-	$\frac{1}{24}$	-	$\frac{1}{24}$	-	-	$\frac{1}{24}$	-	-	-	-	-		
V_0	V_0	100	$\frac{1}{108}$	$\frac{1}{216}$	-	-	$\frac{1}{72}$	$\frac{1}{144}$	$\frac{1}{144}$	-	$\frac{1}{72}$	-	$\frac{1}{72}$	-	-	$\frac{1}{72}$	-	-	-	-	-		
P	V_0	110	-	$\frac{1}{18}$	$\frac{1}{27}$	$\frac{2}{27}$	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{1}{108}$	$\frac{2}{27}$	$\frac{1}{6}$	$\frac{1}{18}$	$\frac{1}{54}$	$\frac{1}{54}$	$\frac{2}{27}$	$\frac{1}{6}$	$\frac{1}{9}$	$\frac{4}{27}$	$\frac{1}{27}$	$\frac{1}{27}$	$\frac{1}{3}$		
V_0	V_0	120	$\frac{5}{27}$	$\frac{5}{54}$	-	-	$\frac{5}{18}$	$\frac{5}{36}$	$\frac{5}{36}$	-	$\frac{5}{18}$	-	$\frac{5}{18}$	-	-	$\frac{5}{18}$	-	-	-	-	-		
							1						1						1				1

Table 3: Transition intensities for the coupling of vector mesons to meson pairs.

			flavour channels and totals for S																					
			SU_3 -octet members															SU_3 singlets						
decay products			isotriplets					isodoublets					isoscalars											
A	B	rel.	(t)					(d)					(8)					(1)						
		LSN	tt	dd	$t8$	$t1$	T	td	$d8$	$d1$	T	tt	dd	88	11	T	tt	dd	88	11	T			
P	P	001	-	$\frac{1}{72}$	$\frac{1}{108}$	$\frac{1}{54}$	$\frac{1}{24}$	$\frac{1}{48}$	$\frac{1}{432}$	$\frac{1}{54}$	$\frac{1}{24}$	$\frac{1}{72}$	$\frac{1}{216}$	$\frac{1}{216}$	$\frac{1}{54}$	$\frac{1}{24}$	$\frac{1}{72}$	$\frac{1}{54}$	$\frac{1}{216}$	$\frac{1}{216}$	$\frac{1}{24}$			
P	P'	000	-	$\frac{1}{144}$	$\frac{1}{216}$	$\frac{1}{108}$	$\frac{1}{48}$	$\frac{1}{96}$	$\frac{1}{864}$	$\frac{1}{108}$	$\frac{1}{48}$	$\frac{1}{144}$	$\frac{1}{432}$	$\frac{1}{432}$	$\frac{1}{108}$	$\frac{1}{48}$	$\frac{1}{144}$	$\frac{1}{108}$	$\frac{1}{432}$	$\frac{1}{432}$	$\frac{1}{48}$			
V_0	V_0	001	-	$\frac{1}{216}$	$\frac{1}{324}$	$\frac{1}{162}$	$\frac{1}{72}$	$\frac{1}{144}$	$\frac{1}{1296}$	$\frac{1}{162}$	$\frac{1}{72}$	$\frac{1}{216}$	$\frac{1}{648}$	$\frac{1}{648}$	$\frac{1}{162}$	$\frac{1}{72}$	$\frac{1}{216}$	$\frac{1}{162}$	$\frac{1}{648}$	$\frac{1}{648}$	$\frac{1}{72}$			
V_0	V_0'	000	-	$\frac{1}{432}$	$\frac{1}{648}$	$\frac{1}{324}$	$\frac{1}{144}$	$\frac{1}{288}$	$\frac{1}{2592}$	$\frac{1}{324}$	$\frac{1}{144}$	$\frac{1}{432}$	$\frac{1}{1296}$	$\frac{1}{1296}$	$\frac{1}{324}$	$\frac{1}{144}$	$\frac{1}{432}$	$\frac{1}{324}$	$\frac{1}{1296}$	$\frac{1}{1296}$	$\frac{1}{144}$			
V_0	V_2	000	-	$\frac{5}{108}$	$\frac{5}{162}$	$\frac{5}{81}$	$\frac{5}{36}$	$\frac{5}{72}$	$\frac{5}{648}$	$\frac{5}{81}$	$\frac{5}{36}$	$\frac{5}{108}$	$\frac{5}{324}$	$\frac{5}{324}$	$\frac{5}{81}$	$\frac{5}{36}$	$\frac{5}{108}$	$\frac{5}{81}$	$\frac{5}{324}$	$\frac{5}{324}$	$\frac{5}{36}$			
U	U	000	-	$\frac{1}{144}$	$\frac{1}{216}$	$\frac{1}{108}$	$\frac{1}{48}$	$\frac{1}{96}$	$\frac{1}{864}$	$\frac{1}{108}$	$\frac{1}{48}$	$\frac{1}{144}$	$\frac{1}{432}$	$\frac{1}{432}$	$\frac{1}{108}$	$\frac{1}{48}$	$\frac{1}{144}$	$\frac{1}{108}$	$\frac{1}{432}$	$\frac{1}{432}$	$\frac{1}{48}$			
S	S	000	-	$\frac{1}{48}$	$\frac{1}{72}$	$\frac{1}{36}$	$\frac{1}{16}$	$\frac{1}{32}$	$\frac{1}{288}$	$\frac{1}{36}$	$\frac{1}{16}$	$\frac{1}{48}$	$\frac{1}{144}$	$\frac{1}{144}$	$\frac{1}{36}$	$\frac{1}{16}$	$\frac{1}{48}$	$\frac{1}{36}$	$\frac{1}{144}$	$\frac{1}{144}$	$\frac{1}{16}$			
T	T	000	-	$\frac{1}{36}$	$\frac{1}{54}$	$\frac{1}{27}$	$\frac{1}{12}$	$\frac{1}{24}$	$\frac{1}{216}$	$\frac{1}{27}$	$\frac{1}{12}$	$\frac{1}{36}$	$\frac{1}{108}$	$\frac{1}{108}$	$\frac{1}{27}$	$\frac{1}{12}$	$\frac{1}{36}$	$\frac{1}{27}$	$\frac{1}{108}$	$\frac{1}{108}$	$\frac{1}{12}$			
P	T	110	-	$\frac{1}{18}$	$\frac{1}{27}$	$\frac{2}{27}$	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{1}{108}$	$\frac{2}{27}$	$\frac{1}{6}$	$\frac{1}{18}$	$\frac{1}{54}$	$\frac{1}{54}$	$\frac{2}{27}$	$\frac{1}{6}$	$\frac{1}{18}$	$\frac{2}{27}$	$\frac{1}{54}$	$\frac{1}{54}$	$\frac{1}{6}$			
V_0	U	110	-	$\frac{1}{18}$	$\frac{1}{27}$	$\frac{2}{27}$	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{1}{108}$	$\frac{2}{27}$	$\frac{1}{6}$	$\frac{1}{18}$	$\frac{1}{54}$	$\frac{1}{54}$	$\frac{2}{27}$	$\frac{1}{6}$	$\frac{1}{18}$	$\frac{2}{27}$	$\frac{1}{54}$	$\frac{1}{54}$	$\frac{1}{6}$			
V_0	V_0	220	-	$\frac{5}{54}$	$\frac{5}{81}$	$\frac{10}{81}$	$\frac{5}{18}$	$\frac{5}{36}$	$\frac{5}{324}$	$\frac{10}{81}$	$\frac{5}{18}$	$\frac{5}{54}$	$\frac{5}{162}$	$\frac{5}{162}$	$\frac{10}{81}$	$\frac{5}{18}$	$\frac{5}{54}$	$\frac{10}{81}$	$\frac{5}{162}$	$\frac{5}{162}$	$\frac{5}{18}$			
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